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10CS42

**Fourth Semester B.E. Degree Examination, June/July 2015**  
**Graph Theory and Combinatorics**

Time: 3 hrs.

Max. Marks:100

**Note: Answer any FIVE full questions, selecting at least TWO questions from each part.**

**PART – A**

- 1 a. For the following graph determine,
- A walk from b to d that is not a trail
  - A b-d trail that is not a path
  - A path from b to d
  - A closed walk from b to b that is not a circuit
  - A circuit from b to b that is not a cycle
  - A cycle from b to b.

(06 Marks)

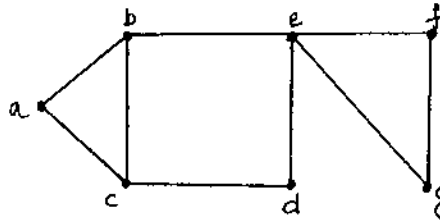


Fig.Q1(a)

- b. Define subgraph, spanning subgraph, induced subgraph and complete graph with example. (07 Marks)
- c. Prove that the undirected graph  $G = (V, E)$  has an Euler circuit if and only if  $G$  is connected and every vertex in  $G$  has even degree. (07 Marks)
- 2 a. Define planar graph and prove that the following Petersen graph is nonplanar using Kuratowski's theorem. (06 Marks)

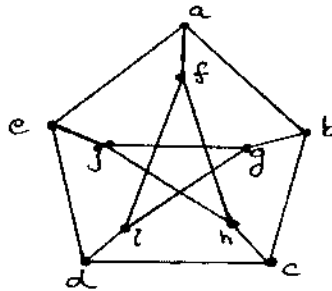


Fig.Q2(a)

- b. Prove that in a complete graph with  $n$ -vertices, where  $n$  is an odd number  $\geq 3$ , there are  $(n-1)/2$  edge-disjoint Hamiltonian cycles. (07 Marks)
- c. Find the chromatic polynomial for the following graph. (07 Marks)

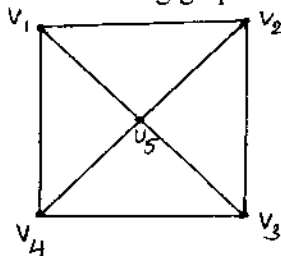


Fig.2Q(c)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
 2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

- 6 a. Find the number of integers between 1 to 10,000 inclusive, which are divisible by none of 5, 6 or 8. (06 Marks)
- b. Determine in how many ways can the letters in the word ARRANGEMENT be arranged so that there are exactly two pairs of consecutive identical letters. (07 Marks)
- c. i) Find the rook polynomial for the shaded chessboard

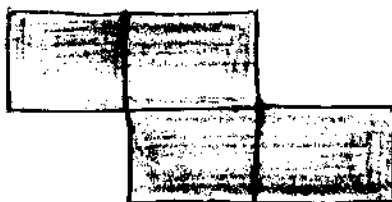


Fig. Q6(c)(i)

- ii) Let  $A = \{1, 2, 3, 4\}$  and  $B = \{u, v, w, x, y, z\}$ . How many one to one functions  $f: A \rightarrow B$  satisfy none of the following conditions :  
 $C_1: f(1) = u$  or  $v$  ;  $C_2: f(2) = w$  ;  $C_3: f(3) = w$  or  $x$  ;  $C_4: f(4) = x, y$  or  $z$ . (07 Marks)
- 7 a. Find the coefficient of  $x^{15}$  in  $\frac{(1+x)^4}{(1-x)^4}$ . (06 Marks)
- b. A ship carries 48 flags, 12 each of the colors red, white, blue and black. Twelve of these flags are placed on a vertical pole in order to communicate a signal to other ships. Determine, how many of these signals have at least three white flags or no white flags at all. (07 Marks)
- c. Find the formula to express :  $0^2 + 1^2 + 2^2 + \dots + n^2$  as a function of  $n$  using summation on operator. (07 Marks)
- 8 a. Solve the recurrence relation  $F_{n+2} = F_{n+1} + F_n$  where  $n \geq 0$  and  $F_0 = 0$  and  $F_1 = 1$ . (06 Marks)
- b. i) A bank pays 6% interest compounded quarterly. If Laura invests \$ 100 then how many months must she wait for her money to double?
- ii) The number of bacteria in a culture is 1000 and this number increases 250% every 2 hours. Use a recurrence relation to determine the number of bacteria present after one day. (07 Marks)
- c. Solve the recurrence relation :  $a_{n+2} - 5a_{n+1} + 6a_n = 2$ ,  $n \geq 0$ ,  $a_0 = 3$ ,  $a_1 = 7$  using method of generating functions. (07 Marks)

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